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Investigating the Time Dependent Behavior of Thermoplastic Polymers under Tensile Load

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Summary: The relationship between the results of the tensile and the stress relaxation tests of polypropylene specimens were analyzed and an attempt was made to find a way to estimate the former from the latter based on the measurements and the theory of linear viscoelasticity. The mechanical response of real polymers are basically of nonlinear character, therefore their behavior patterns do not meet the idealized (linear) ones. Experiments were performed on poly(propylene) (PP) as a test material and the stress relaxation behavior, as well as the linear elastic and linear viscoelastic approximation of the tensile load-time curve were analyzed. To demonstrate the applicability of our idea and to perform the numerical calculations we have chosen a flexible function with three parameters to realize the nonlinear behavior.

Keywords: mechanical properties; poly(propylene) (PP); stress relaxation; tensile load-time curve; viscoelastic properties

Introduction

The use of thermoplastics in structural applications demands accurate design data that spans appropriate ranges of stress, strain rate, time and temperature. Despite polymer viscoelasticity, materials design data traditionally have neglected this phenomenon. In an effort to address this shortcoming, new methodologies are continually being developed [1–4]. A number of theoretical and experimental studies and books have mainly dealt with the investigation of the relaxation phenomena and in some cases with the relationships between the results of different tests in order to estimate or predict the short and long term behavior of polymers under different conditions [5–7]. The stress relaxation (SR) behavior of thermoplastics has been studied in several theoretical and experimental investigations such as those by Urzumtsev and Maksimov [6], Retting [7], Wortmann and Shultz [8], Andreassen [9], as well as Baeurle et. al. [10]. Grzywinski and Woodford [3] and Sudduth [2] have dealt with the relationships between creep, SR and constant strain rate data. This publication is the first part of a multi-part series in which we investigate the possibilities to predict long-term mechanical behavior – stress relaxation, creep – from short-term experiments (e.g. tensile tests) supposing that during tensile tests the specimen lives its life accelerated – until it fails.

The relationship between the long term and short term behavior is predicted based on the Theory of Linear Viscoelasticity. Nonlinear effects are planned to take into account with the help of variable transformation as it can be found in various well-known applications – e.g. the WLF or Arrhenius Equations in time-temperature equivalence.

This article deals with the somewhat simpler inverse problem, namely to predict the short term behavior from the long term one. Solving the inverse problem facilitates the finding of the proper transformations to correct for the nonlinear effects. For this

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reason we have chosen a not too long testing time (400 s), temporarily. Two methods were compared for the estimation of the real tensile load response by using the viscoelastic response given to the real relaxation stimulus: a simple cross-plotting technique described in [3] and a new method proposed by the authors, based on linear and non-linear viscoelastic (LVE and NLVE) responses. A flexible function with three parameters was chosen to realize the nonlinear behavior.

Summary of the Theoretical Considerations

Figure 1 shows the general scheme of mechanical tests where S is the specimen as a black box system, X is the input (stimulus) and Y is the response of the material.

If S is an operator characterizing the material the response can be obtained as follows:

$$Y(t) = S[X](t) \tag{1}$$

In practice operator S is often considered to be continuous and linear which leads to the following convolution integral:

$$Y(t) = \int_0^t W(t-u)dX(u) \tag{2}$$

where W is a characteristic function of the material. The usual inputs (X) are the strain (ε) or the stress (σ).

In case of tensile tests usually force (F)-elongation (Δl) curves are recorded instead of stress $\sigma(t)$-strain $\varepsilon(t)$ curves. The relationships between them are:

$$F(t) = A_0\sigma(t) \tag{3}$$

$$\Delta l(t) = l_0\varepsilon(t) \tag{4}$$

where A_0 and l_0 are the cross sectional area and the gauge length of the unloaded specimen, respectively.

Figure 1.
General scheme of mechanical tests.

Let $F_1(t)$ be the relaxation curve as a response to the following real stimulus

$$\begin{aligned}\Delta l_1(t) &= l_0\varepsilon_1(t)\\ &= l_0\dot{\varepsilon}_0 t(l(t) - l(t-t_0))\\ &\quad + l_0\varepsilon_0 l(t-t_0)\end{aligned} \tag{5}$$

and $F_2(t)$ be the tensile force-time component of the measured tensile load-strain curve as a response given to the following real stimulus:

$$\Delta l_2(t) = l_0\varepsilon_2(t) = l_0\dot{\varepsilon}_0 t l(t) \tag{6}$$

Taking into account that :

$$\varepsilon_0 = \dot{\varepsilon}_0 t_0 \tag{7}$$

the stimulus according to Equation (5) can be produced as the difference of stimuli according to (8):

$$\begin{aligned}\Delta l_1(t) &= l_0\varepsilon_1(t)\\ &= l_0\dot{\varepsilon}_0 t l(t) - l_0\dot{\varepsilon}_0(t-t_0)l(t-t_0)\\ &= l_0(\varepsilon_2(t) - \varepsilon_2(t-t_0))\end{aligned} \tag{8}$$

The convolution integral in Eq. (2) is a linear operator therefore the response given to a sum of stimuli can be produced as a sum of partial responses given to partial stimuli. Consequently, in case of LVE material behavior, relaxation curve $F_1(t)$ as a response given to real stimulus (6), can be given with the aid of tensile load-time curve $F_2(t)$, as well (Figure 2):

$$F_1(t) = F_2(t) - F_2(t-t_o) \tag{9}$$

The relaxation curve, $F_1(t)$, can be calculated from the load-time curve, $F_2(t)$, and *vice versa*.

On the basis of Eq. (7) the LVE estimation ($F_{12}(t)$)of the real tensile load-time curve ($F_2(t)$) of the given material can be calculated (see Figure 3).

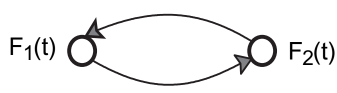

Figure 2.
Calculability of LVE characteristics.

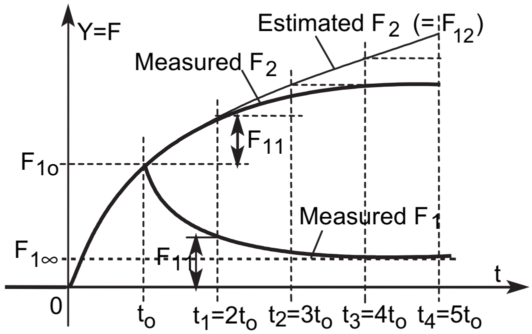

Figure 3.
Estimation of the tensile curve (F_{12}) from the measured stress relaxation curve (F_1).

For example, using sampling time points $t = it_0$ ($i = 1, 2, \ldots$):

$$F_{12}(it_0) = F_1(it_0) + F_{12}((i-1)t_0) \qquad (10)$$

Figure 4 shows the meaning of the various subscripts and illustrates possible NLVE methods for estimating the tensile load-time curve from SR measurement, and *vice versa*.

Experimental

Isotactic PP homopolymer (Tipplen H 543 F from TVK, Hungary) having a MFI (2.16 kg/230 °C) of 4 g/10 min was used for the tests.

Dumbbell specimens were injection molded according to ISO 294-2 Standard on an Arburg Allrounder 320 C 600-250 machine of specimen length 148 mm, width 10 mm and thickness 4 mm. Uniaxial tensile tests and SR tests were performed on a Zwick Z005 tensile testing machine at room temperature. Crosshead speed was 5 mm/min. For the SR tests specimens were stretched up to 0.08, 0.16, 0.24, 0.50, 0.75, 1.00, 1.50, 2.00, 3.00, 5.00 and 7.00 % engineering strains (ε_0). No necking of specimens was observed during the SR tests. Every test was performed on a new specimen. The engineering yield strain was calculated from the crosshead displacement as $\varepsilon_0 = \Delta l / l_0$ where l_0 is the gauge length(=110 mm) at time 0.

Results

Figure 5 shows the results of the tensile and SR tests. In Figure 6, the force is plotted as the function of force rate.

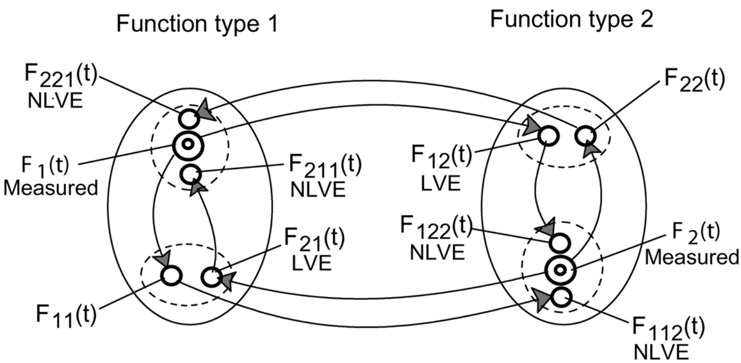

Figure 4.
Possible NLVE estimation ways of the real characteristics using the LVE estimations. (Dashed line circles or ellipses denote the approximation vicinity of the estimated function.)

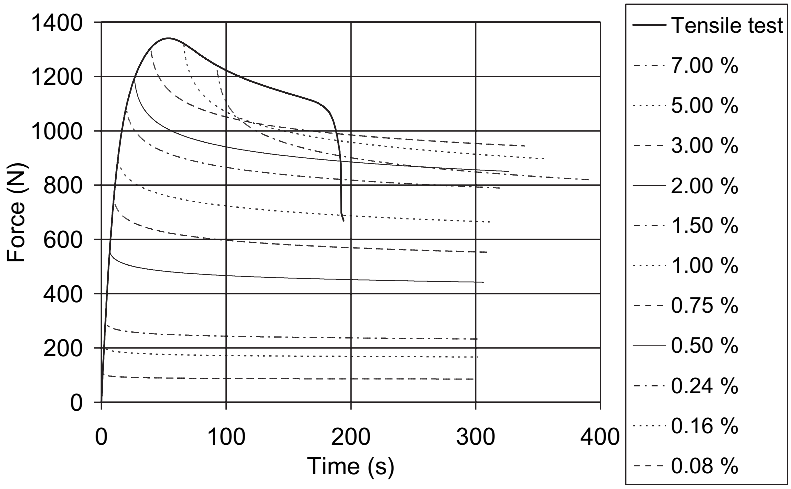

Figure 5.
Results of the tensile and SR tests.

Using these curves iso-force rate pseudo-tensile curves can be constructed. In Figure 7, the so constructed tensile curve is compared to a measured one (both are at 20 N/s iso-force rate).

In Figure 8 and 9, a NLVE estimation (F_{112}) of the tensile load curve (F_2) based on NLVE estimation (F_{11}) of the measured relaxation curve (F_1) can be seen (cf.Figure 4) for two different strains.

In this case the nonlinear operator, S, is realized by a 3-parameter function:

$$F_{11}(t) = F_{11}(t_0) - a(\varepsilon) \times [f_1(t)]^{b(\varepsilon)} e^{c(\varepsilon) f_1(t)} \tag{11}$$

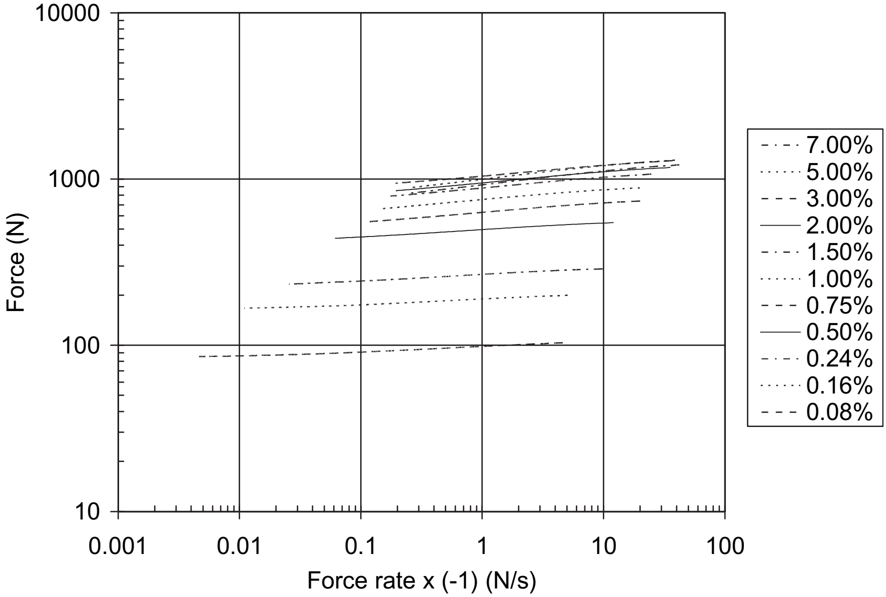

Figure 6.
Force – force rate curves.

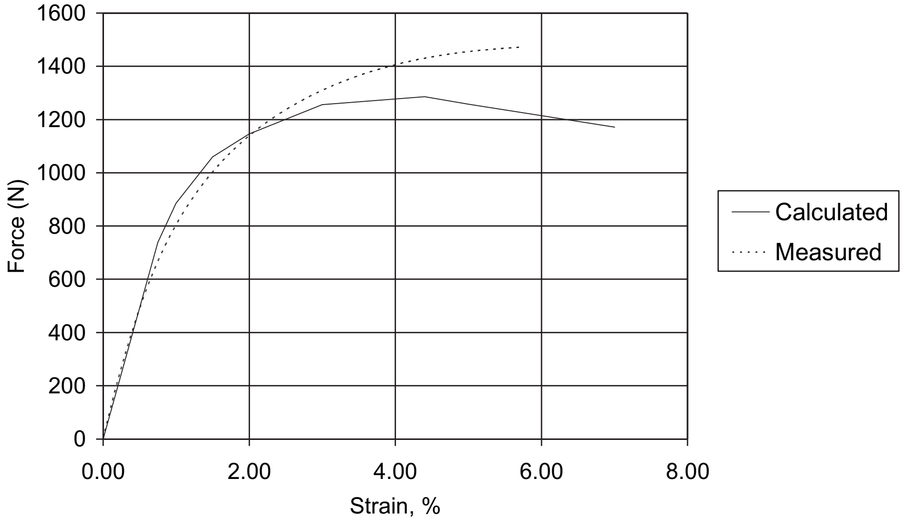

Figure 7.
Measured and calculated iso-force rate (20 N/s) tensile curves.

where $f_1(t) = F_1(t_0) - F_1(t)$ and $t_0 = \varepsilon_0/\dot{\varepsilon}_0$

$$F_{112}(t) = F_{112}(t - t_0) + F_{11}(t) \qquad (12)$$

Discussion

This paper made an attempt to investigate the relationships between the results of the tensile and the stress relaxation tests of poly(propylene) specimens and found a way to estimate the former from the latter based on the measurements and the theory of the linear viscoelasticity used for real stimuli.

In order to demonstrate the applicability of our idea for nonlinear approximation and to perform the numerical calculations we have chosen a flexible function with three parameters to realize the nonlinear

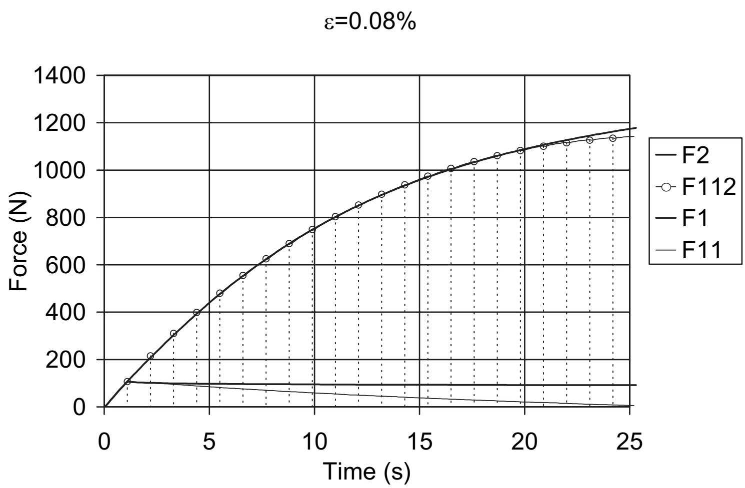

Figure 8.
NLVE estimation of the tensile load-time curve with the relaxation curve at strain level $\varepsilon = \varepsilon_0 = 0.08\%$ ($t_0 = \varepsilon_0/\dot{\varepsilon}_0 = 1.06\,s$).

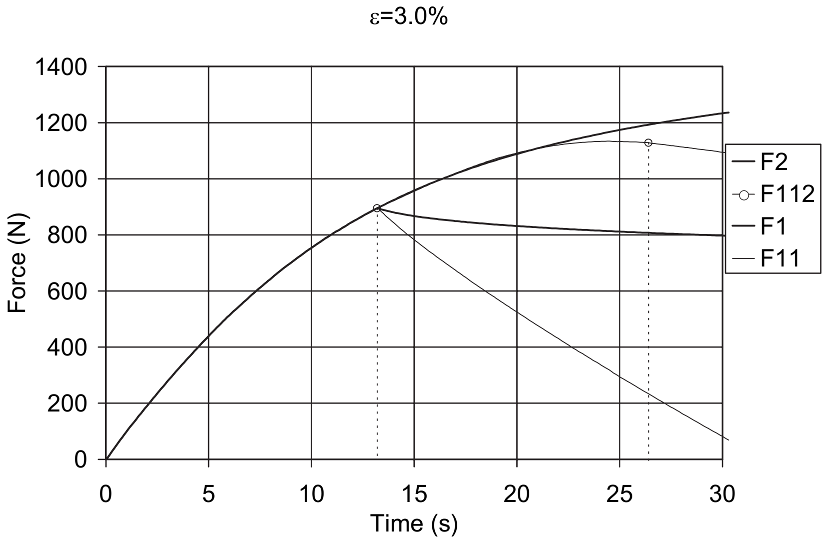

Figure 9.
NLVE estimation of the tensile load-time curve with the relaxation curve at strain level $\varepsilon = \varepsilon_0 = 3.0\% (t_0 = \varepsilon_0 / \dot{\varepsilon}_0 = 13.2\,\mathrm{s})$.

behavior. From the experimental and numerical analysis it was concluded that for the PP material tested the NLVE approximation elaborated here resulted in 1.6 % s (21 s) for the 0.08 % initial strain level and 1.7 % (22 s) for the 3% initial strain level, as a range of good fit (Fig. 8 and 9 respectively).

The range of good fit was only 0.6 % for the iso-force rate (20 N/s) tensile curves (Fig. 7), which means that the NLVE approximation widens the "window" of good fit almost threefold. This method can also be applied to estimate the relaxation curve from the tensile load-time curve.

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